On the Optimal Conflict Resolution for Air Traffic Control

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Introduction

- Formulation as Optimal Control Problem (OCP):
  - Extremal solution (PMP):
    - Unconstrained path;
    - Constrained path of zero length;
    - Constrained path of non-zero length;
  - Conflict Resolution Algorithm;

- Formulation as a Mixed Integer Programming (MIP)

Model of OCP

Motion of aircraft subject to some constraint:

- linear velocity parallel to a fixed axis on the vehicle
- constant non negative linear velocities
- bounded steering radius
- Minimum distance between aircraft

GOAL: given an initial and a final configuration for each aircraft, find the collision free paths of minimum total time.

The OCP

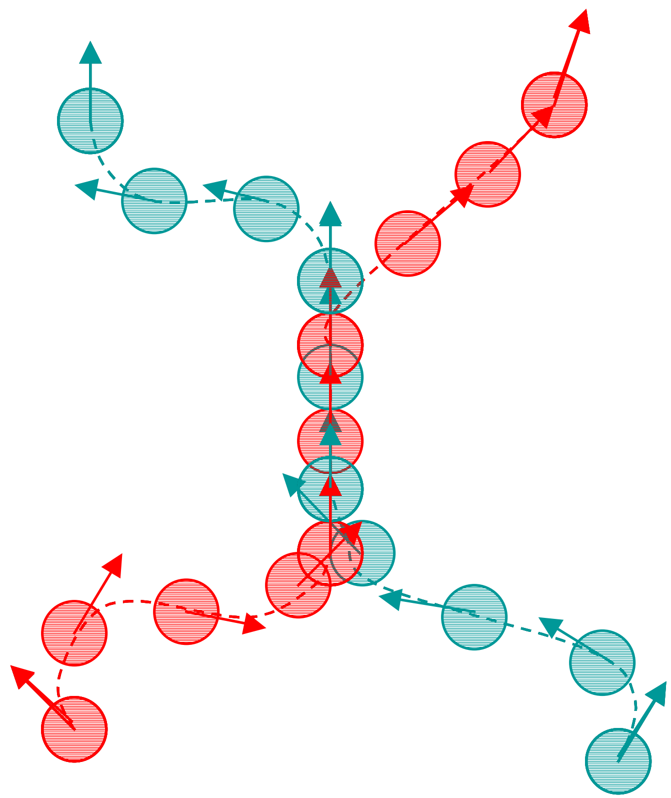
$$
\begin{cases}
\min J & \\
x_i' = u_i \cos\theta_i & \\
y_i' = u_i sin\theta_i & i = 1,\dots,N \\
\theta_i' = \omega_i & \\
|\omega_i| \le \dfrac{u_i}{R_i} & i = 1,\dots,N \\
D_{i,j} >= 0 & \forall t, \quad i, j = 1,\dots,N \\
q_i(T_i^s) = q_i^s, q_i(T_i^g) = q_i^g & i = 1,\dots,N
\end{cases}
$$

collision constraint:

$$
D_{i,j}(t) = \sqrt{(x_i(t) - x_j(t))^2 + (y_i(t) - y_j(t))^2} - d_{i,j} \ge 0
$$

Optimal Solution for OCP

Optimal solution will consist of concatenations of free and constrained arcs, e.g.



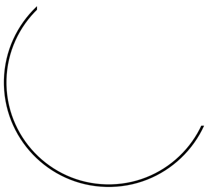
Unconstrained

Constrained

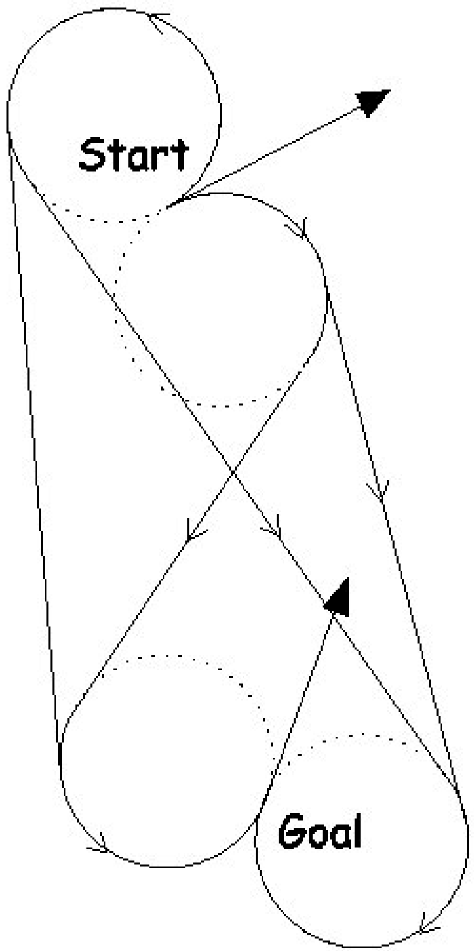
Unconstrained

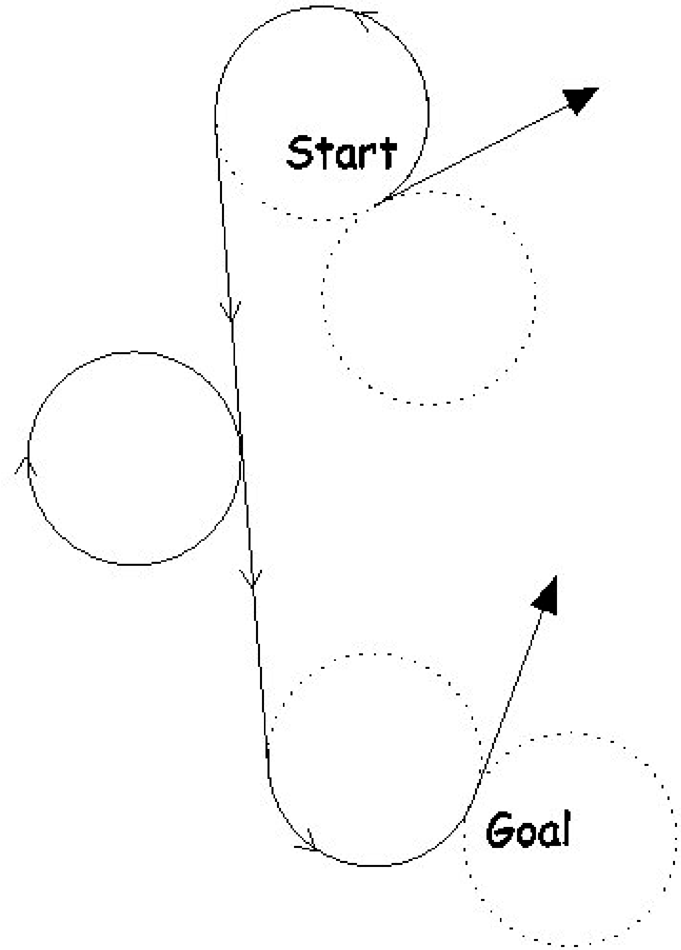
Unconstrained paths for OCP

- Extremal unconstrained path are concatenation of
  - segment — Type S
  - Arc of a circle — Type C

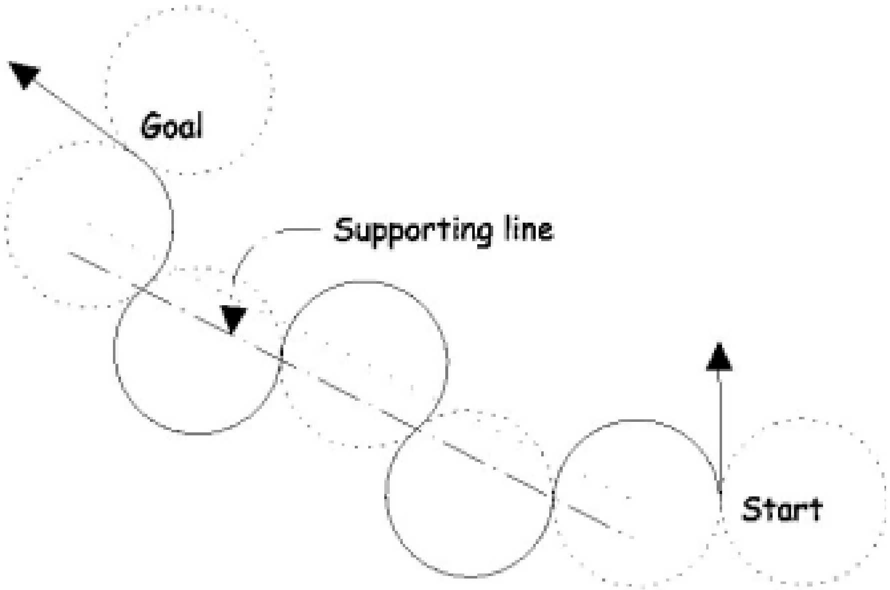


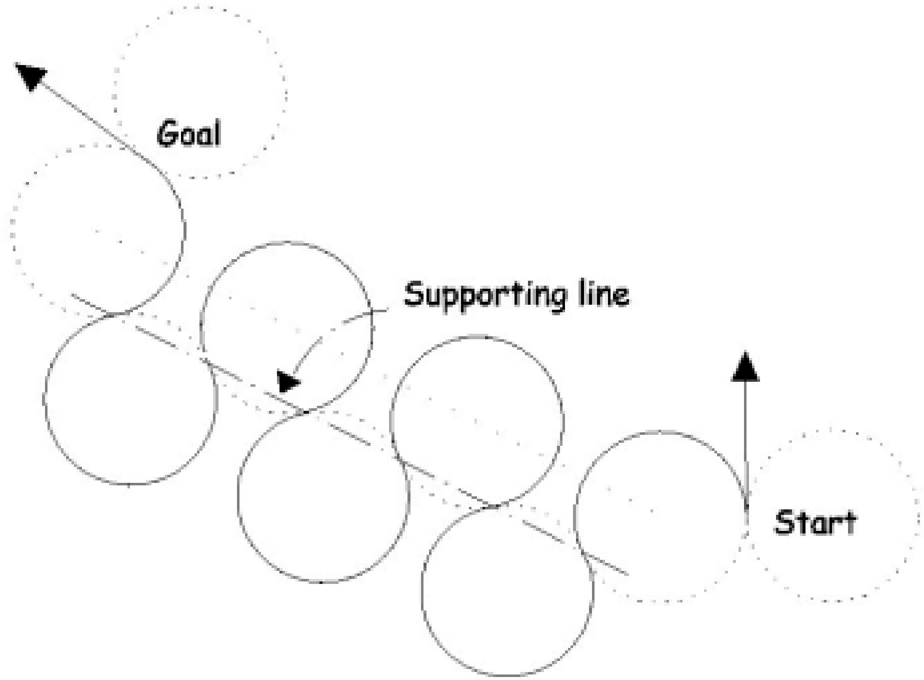
Extremal free paths of type CSC



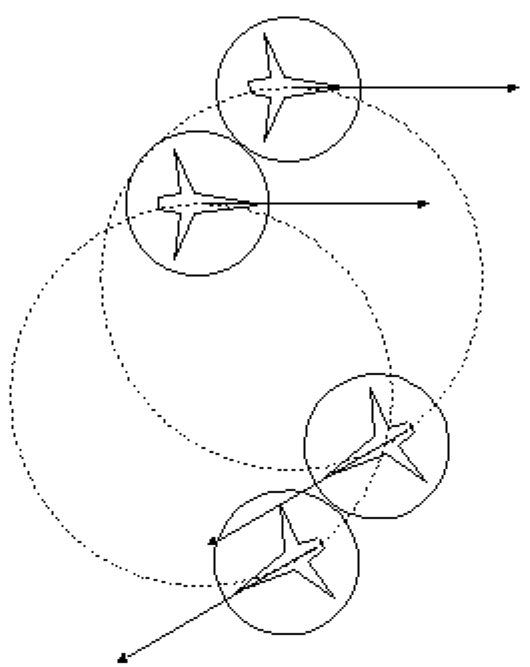


Extremal free paths of type CCC



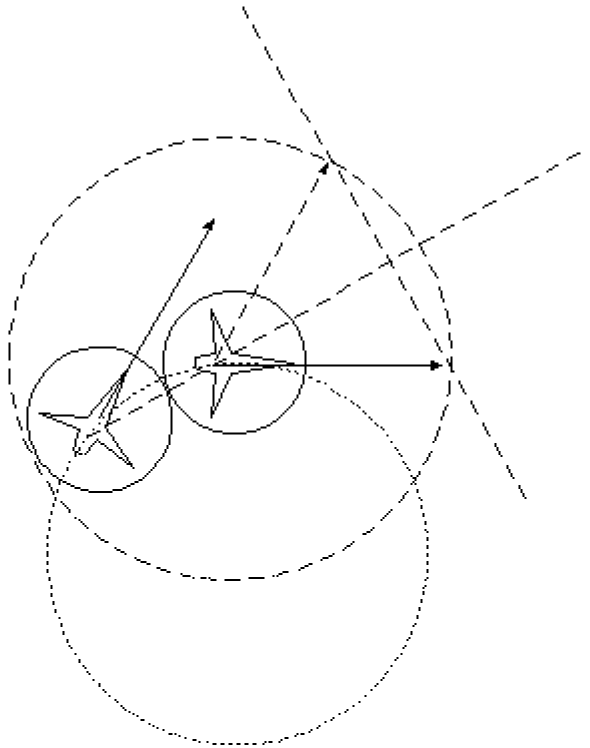


Contact Configuration



a)

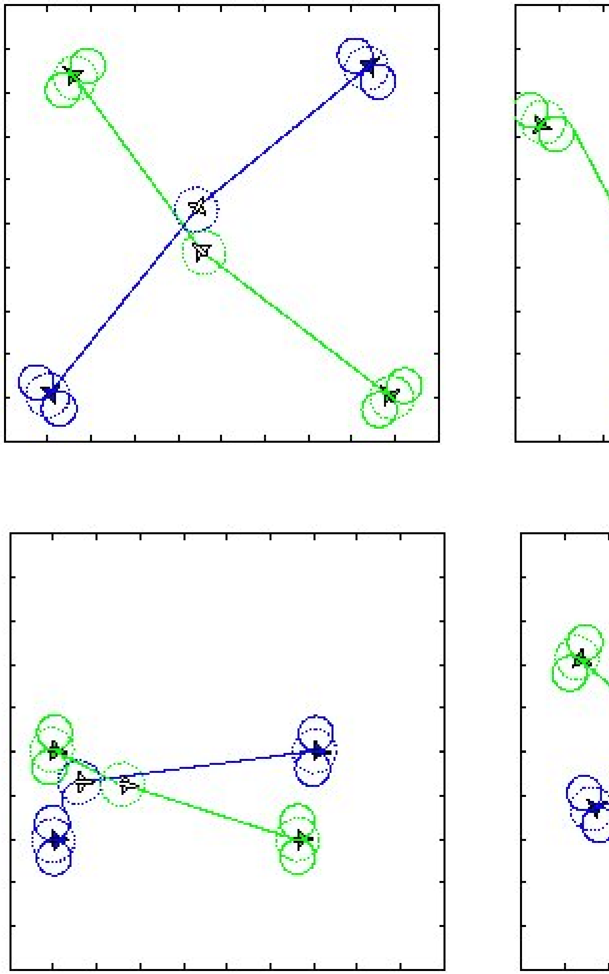
Velocities are parallel and the line joining the two aircraft can only translate



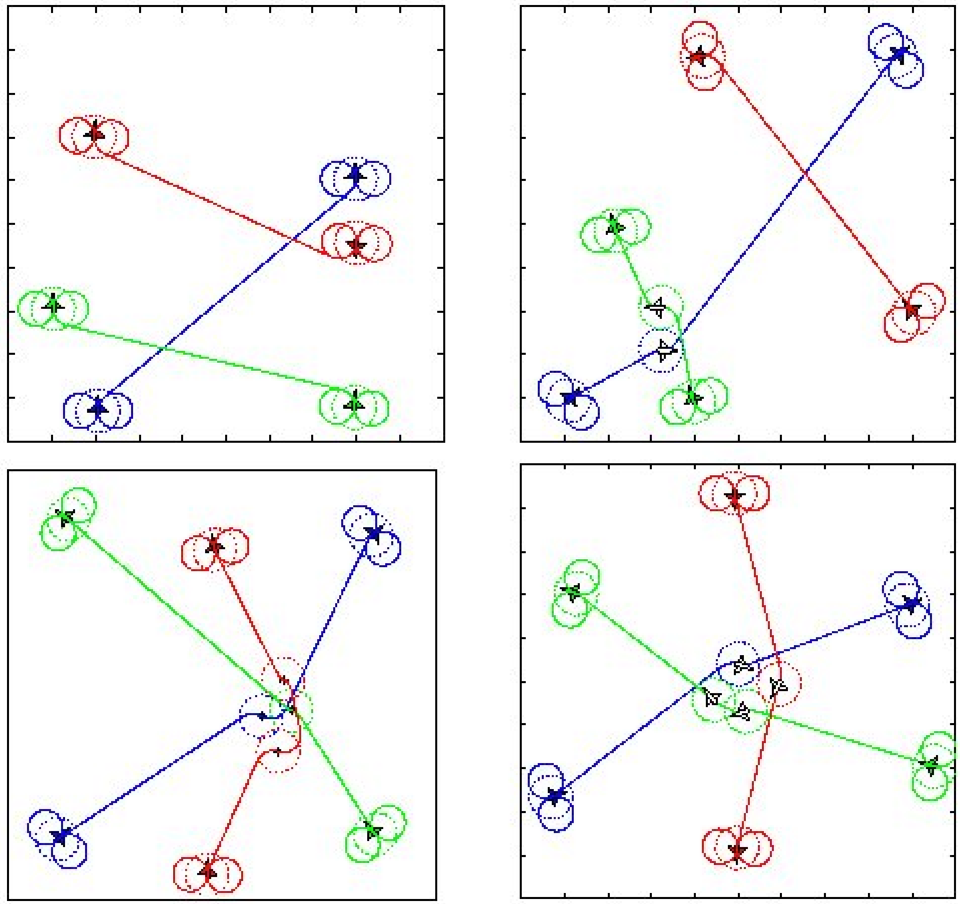
b)

Velocities are symmetric respect to the line joining the two aircraft

Conflict Resolution Algorithm



Conflict Resolution Algorithm



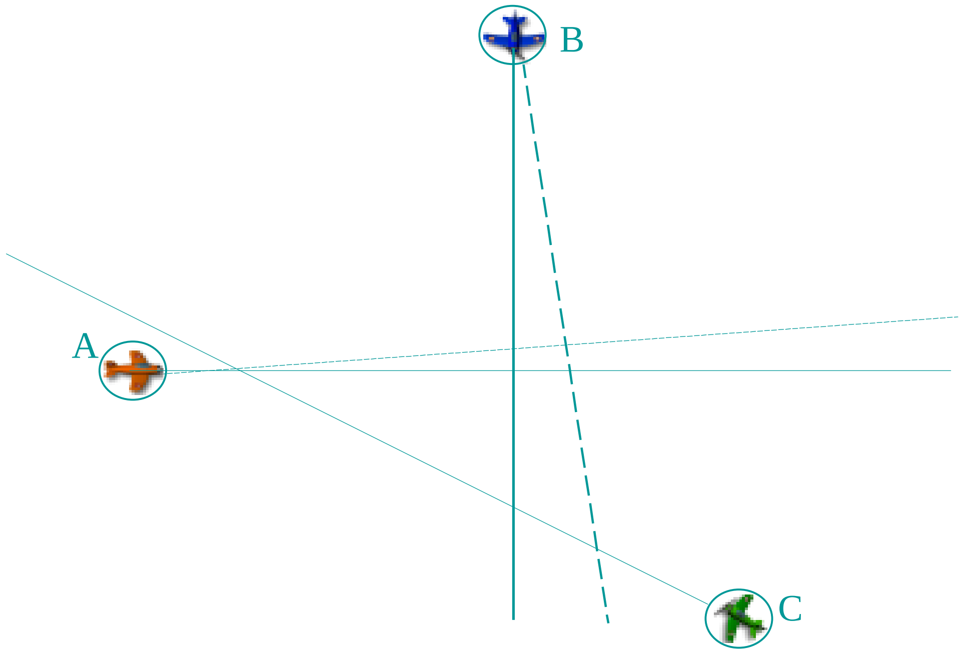
Model of MIP

Motion of aircraft subject to some constraint:

- linear velocity parallel to a fixed axis on the vehicle
- constant non negative linear velocities
- Minimum distance between aircraft
- Maneuver: heading angle instantaneous change

GOAL: given an initial and a final configuration for each aircraft, find a single “minimum” maneuver to avoid all possible conflict.

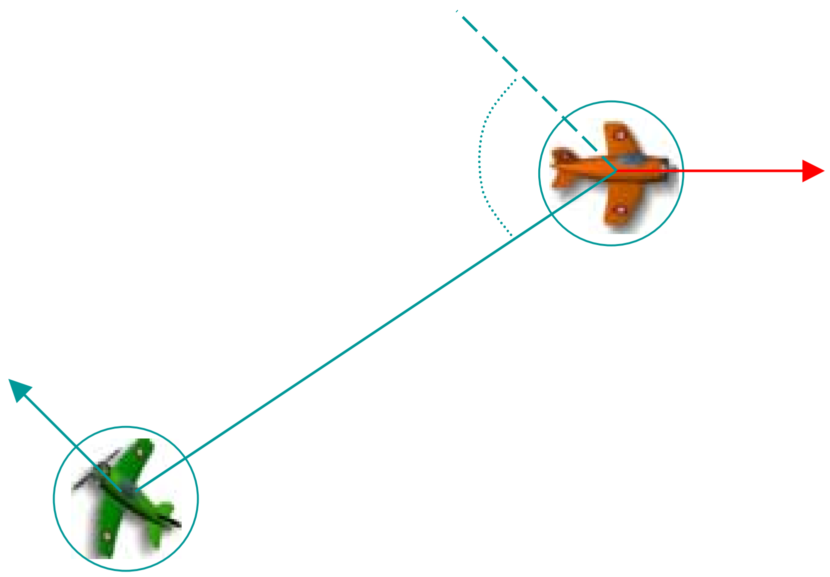
Maneuvre scenario for MIP



Initial Configuration: (x_i, y_i, θ_i)

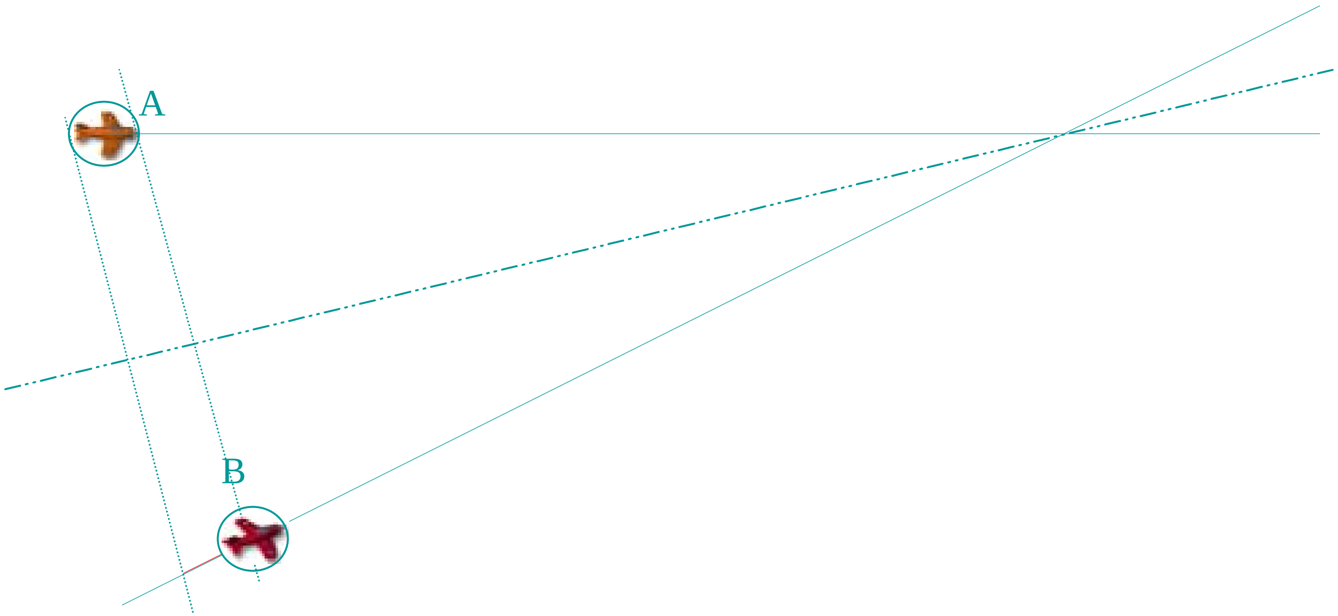
Configuration after maneuver: $(x_i, y_i, \theta_i + p_i)$

Nonintersecting direction of motion



Constraints that are function of (x_i, y_i, θ_i)
and are linear in p_i

Intersecting direction of motion



Constraints that are function of (x_i, y_i, θ_i)

and are linear in p_i

Linear constraint for MIP

$$g_1(x, y, \theta, p) \le b_1$$

or

$$g_2(x, y, \theta, p) \le b_2$$

or

$$g_3(x, y, \theta, p) \le b_3$$

g_i are linear function in p_j

$$g_1(x, y, \theta, p) - f_1 M \le b_1$$
$$g_2(x, y, \theta, p) - f_2 M \le b_2$$
$$g_3(x, y, \theta, p) - f_3 M \le b_3$$
$$f_1 + f_2 + f_3 \le 2$$

M “big” positive number

f_i Boolean variables

The MIP problem

Minimum deviation problem:

$$\min \|p\|_t$$

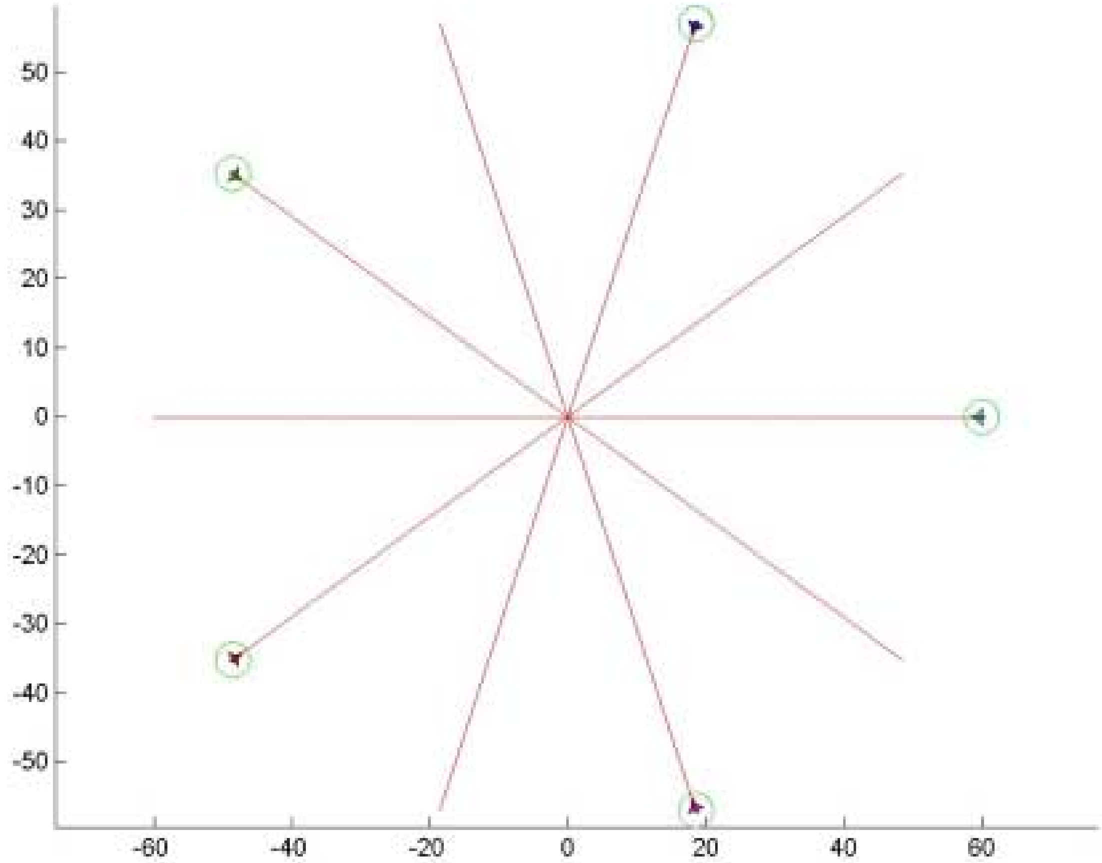
$$A_1 p + A_2 f \le b$$

$$f \quad Boolean$$

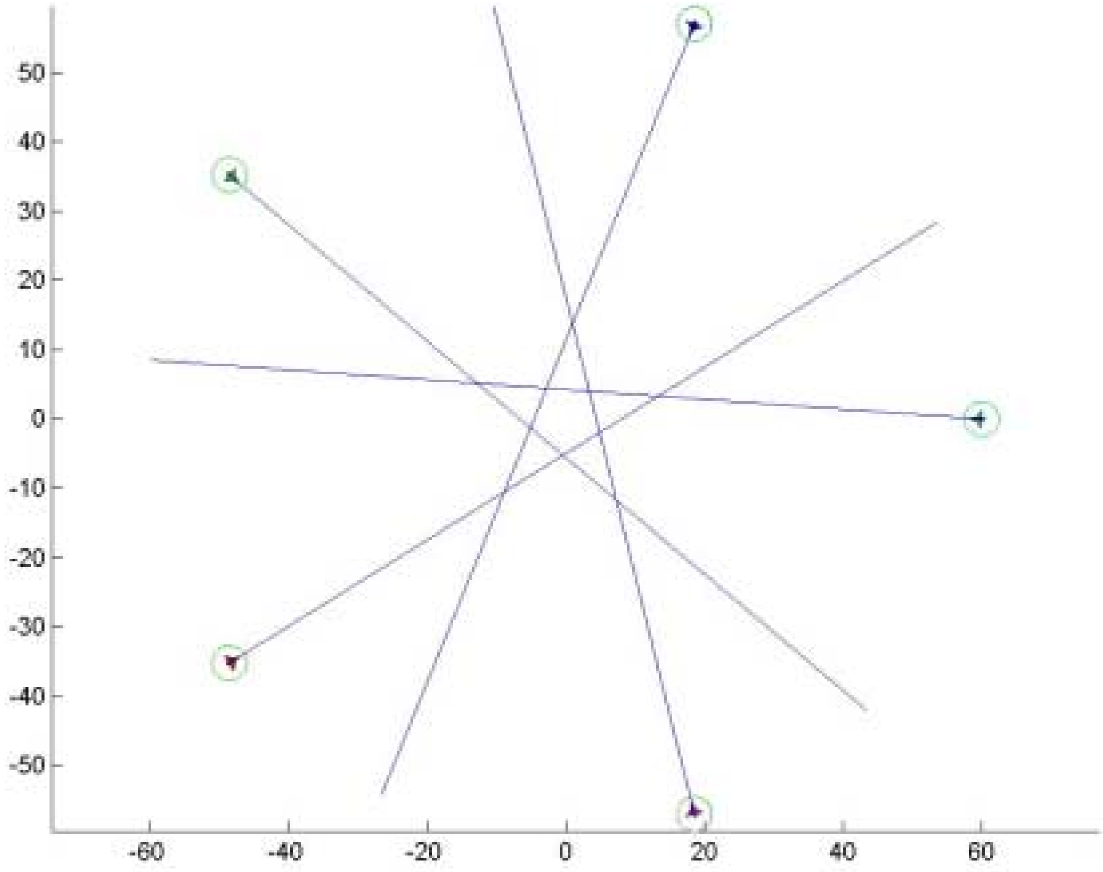
n	Aircraft
$7n^2$	variables
$23n^2$	constraints

- $t = 1 \quad \min \sum_{i=1}^{n} |p_i|$
- $t = \infty \quad \min \max_{i=1,\ldots,n} p_i$

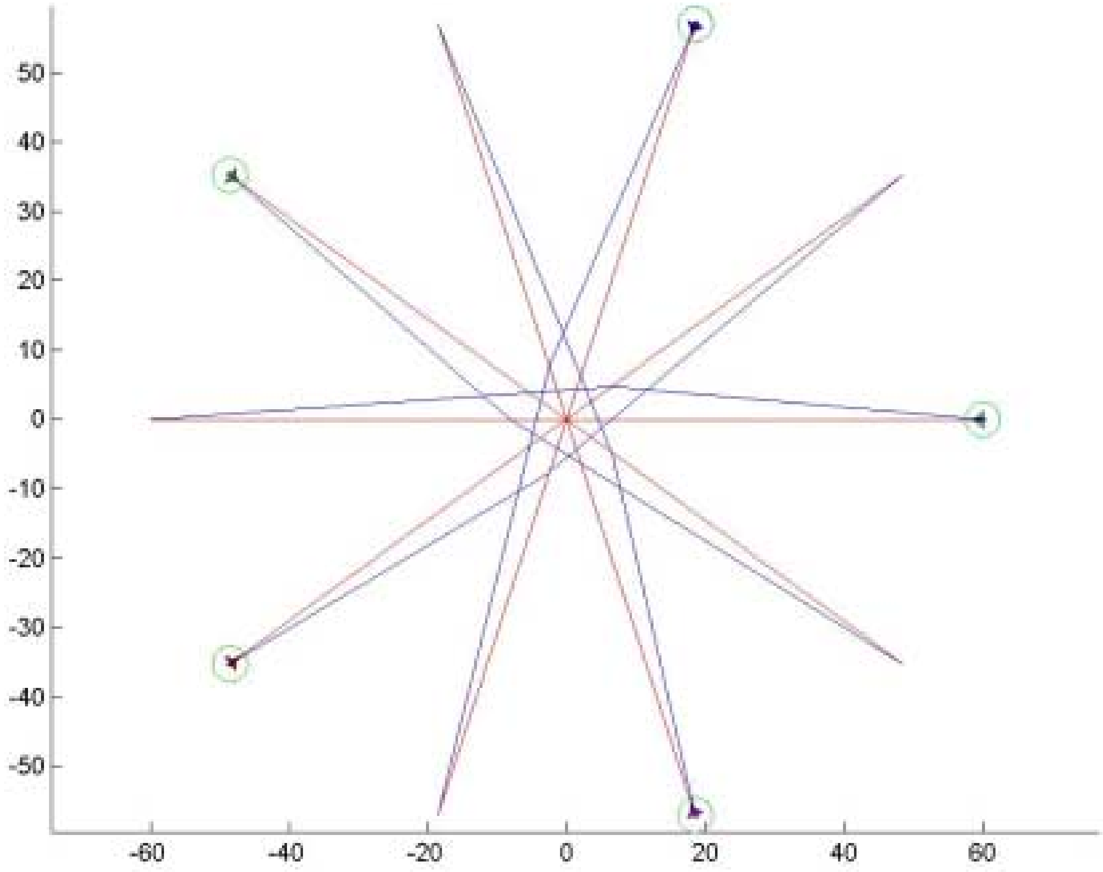
CPLEX Solutions



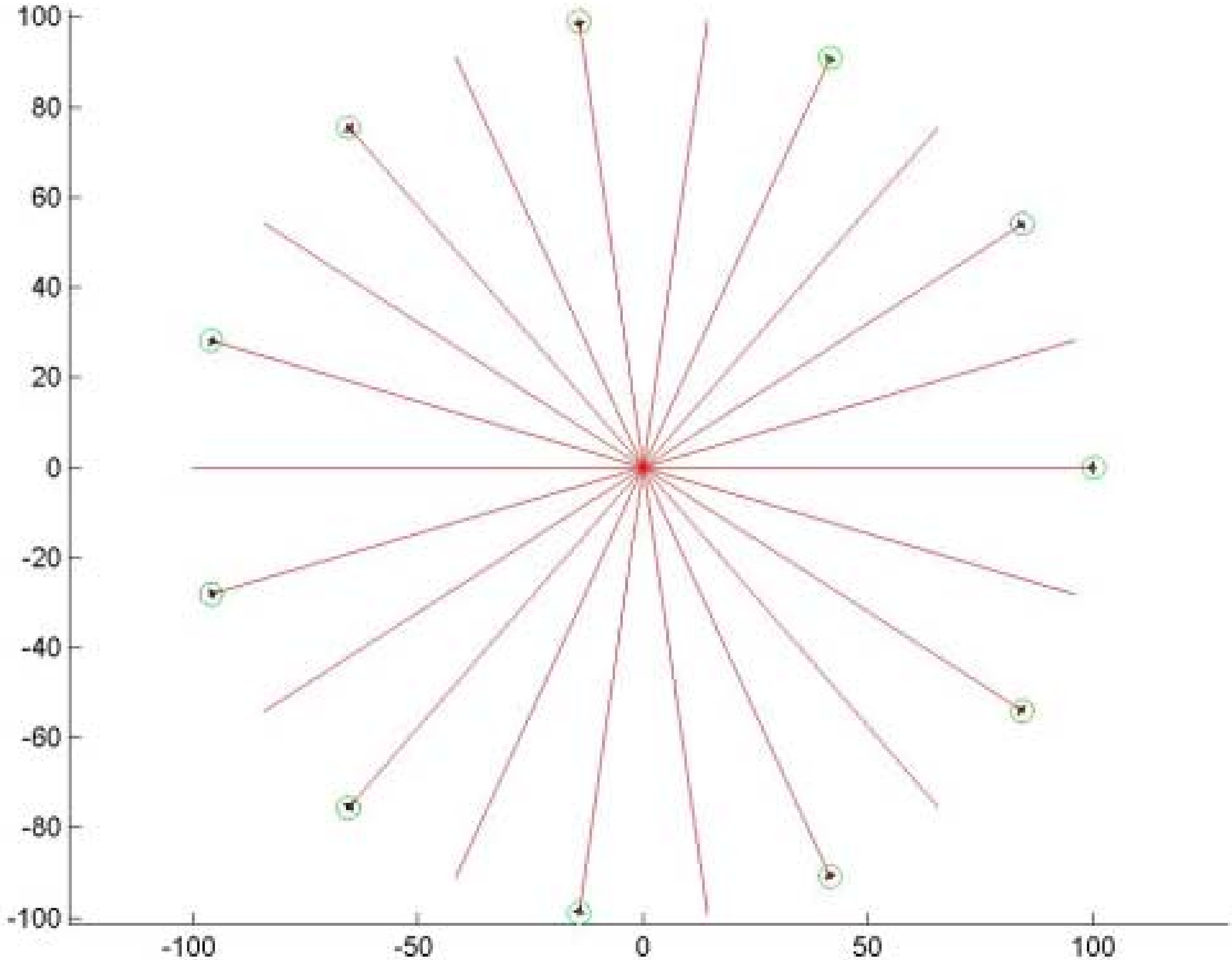
CPLEX Solutions



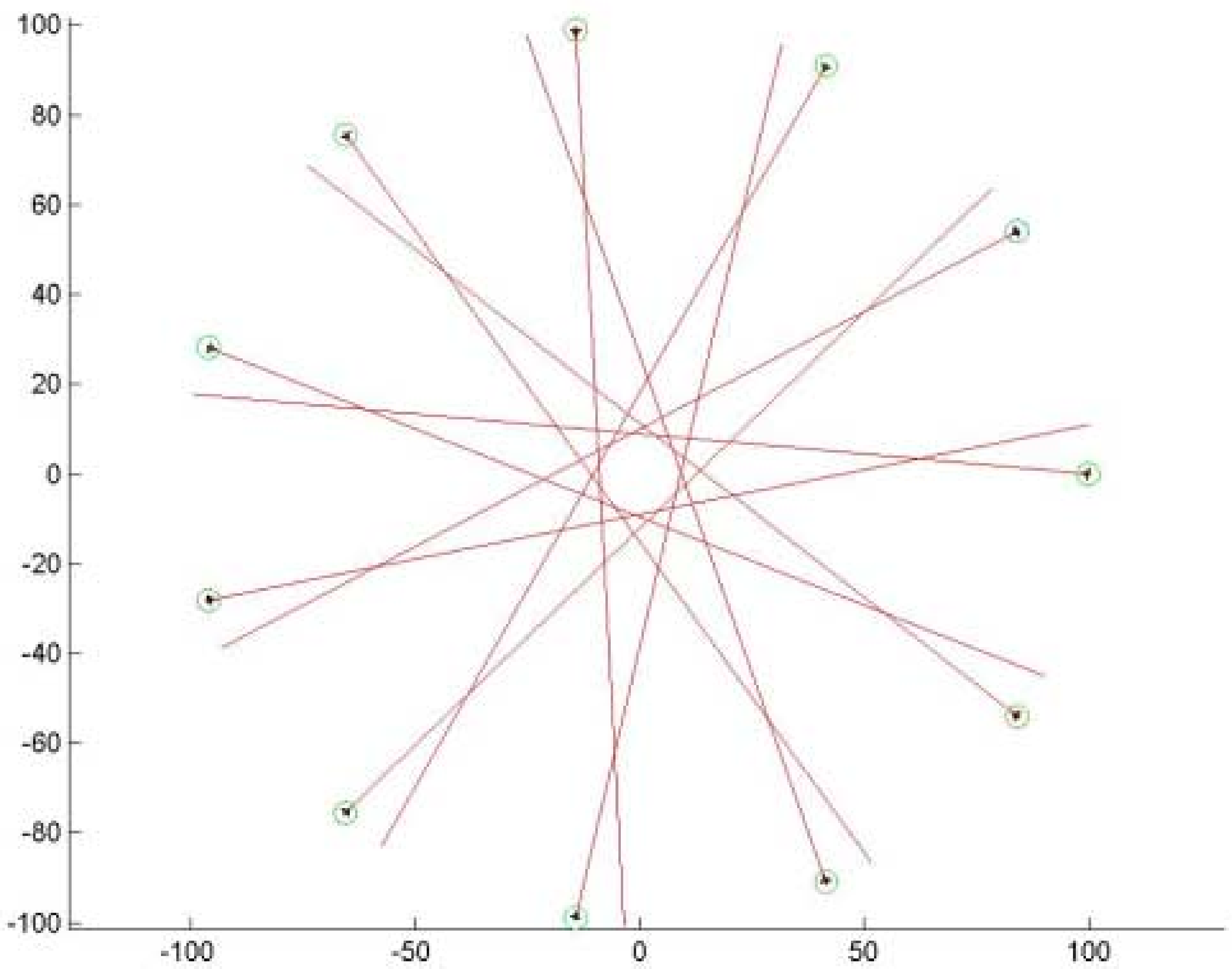
CPLEX Solutions



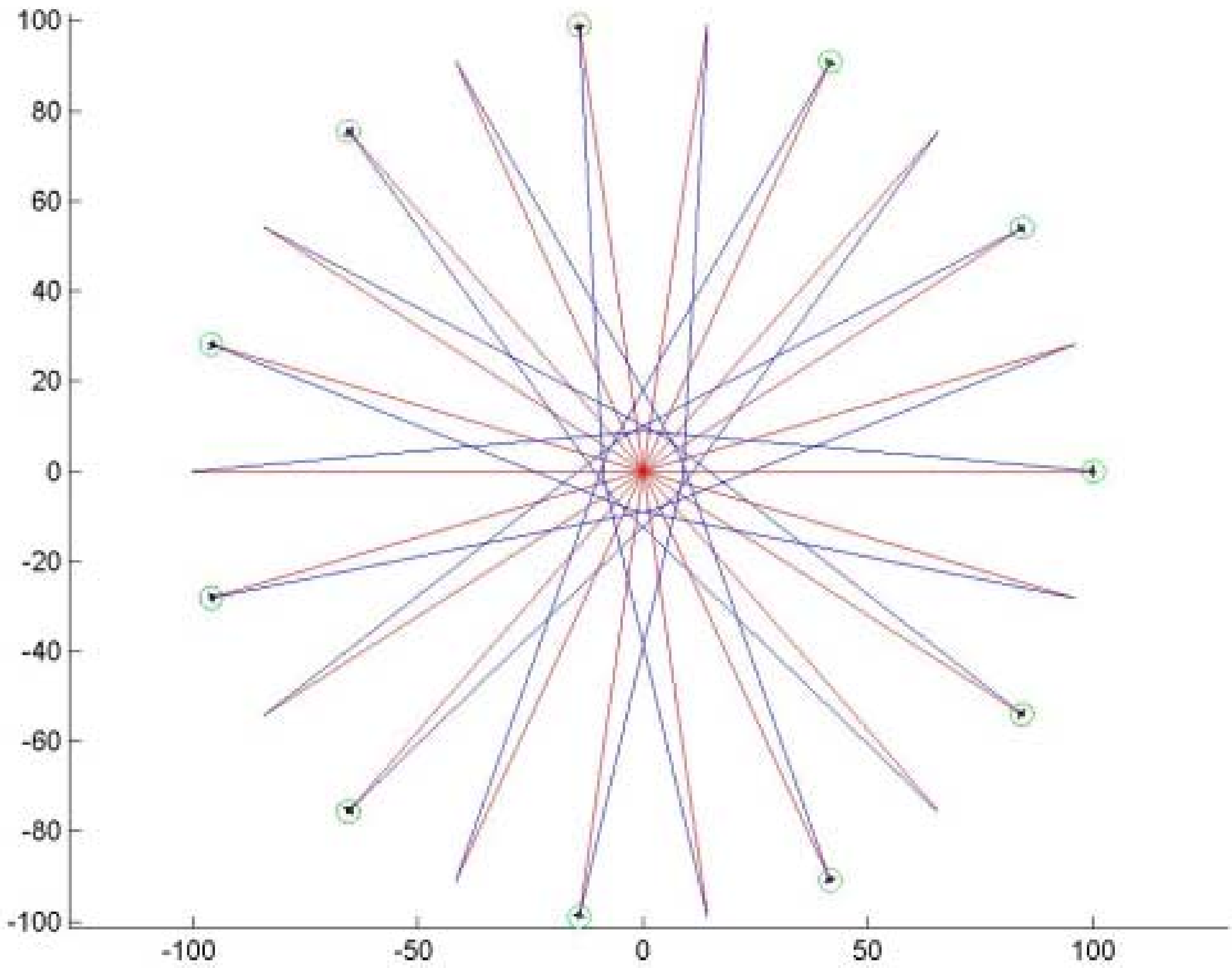
CPLEX Solutions



CPLEX Solutions



CPLEX Solutions



CPLEX Simulation

n (Aircraft)	Time (sec.)	Length (nm)	Delta (nm)
5	0.34	120	0.25
7	1.18	120	0.55
10	5.91	200	0.45
11	10.4	200	0.79